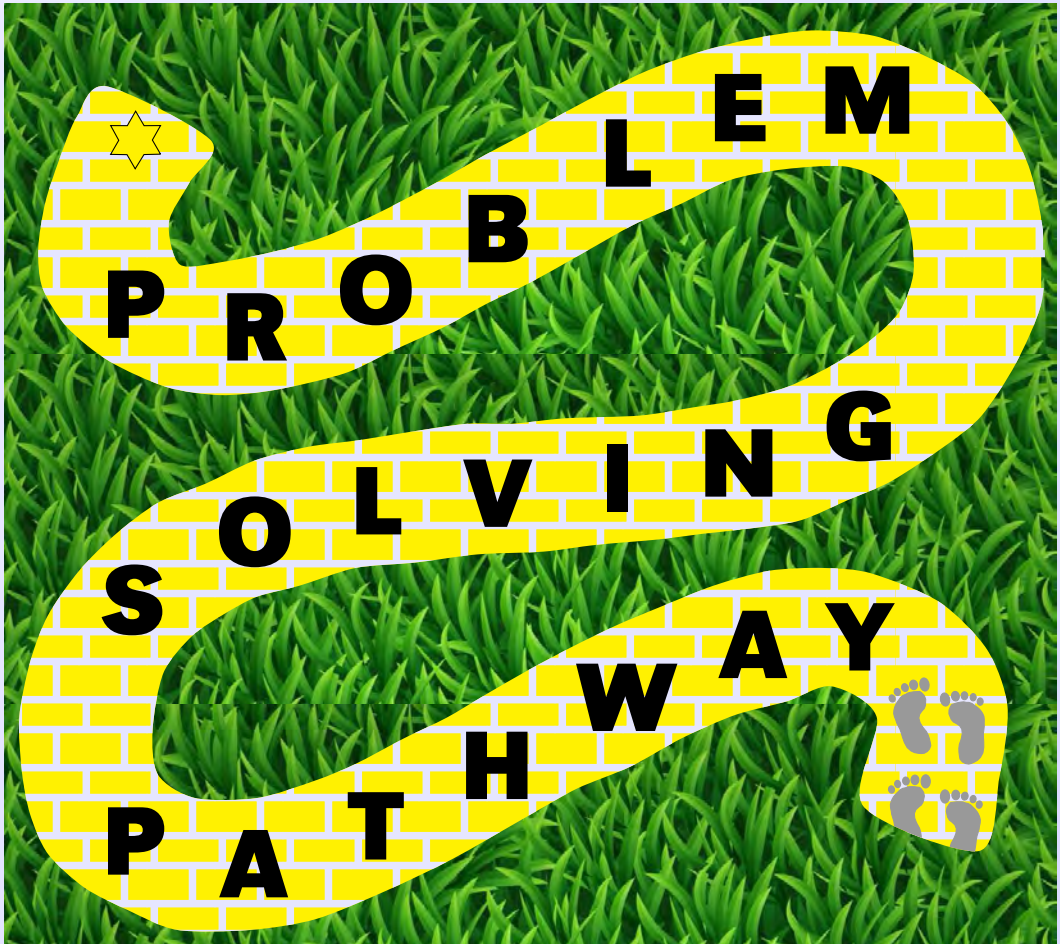
Problem Solving Pathway

Selected Problems



Problem Solving, The Fun Way

Sudoku Strategy

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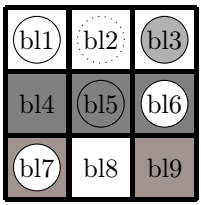
0.1 Sudoku Strategy

Su = *number*, *Doku* = *single* is a logic based number puzzle. Do not confuse it with the similar sounding *Sodoku*, a bacterial disease that causes a form of rat-bite fever.

The objective is to fill numbers 1-9 into a 9×9 grid (81 cells) such that each number appears exactly once in each row, and in each column, and in each of nine 3×3 blocks.

	A	B	C	D	E	F	G	H	I
J	2	6	9	5	7	1	4	3	8
K	7	8	4	3	6	9	2	1	5
L	3	5	1	8	4	2	7	9	6
M	5	3	2	4	1	7	8	6	9
N	6	1	7	2	9	8	3	5	4
O	4	9	8	6	3	5	1	2	7
P	1	2	6	7	5	4	9	8	3
Q	8	7	3	9	2	6	5	4	1
R	9	4	5	1	8	3	6	7	2

Legend:
Nine Rows J-R.
Nine Columns A-I.
Nine Blocks bl1-bl9.



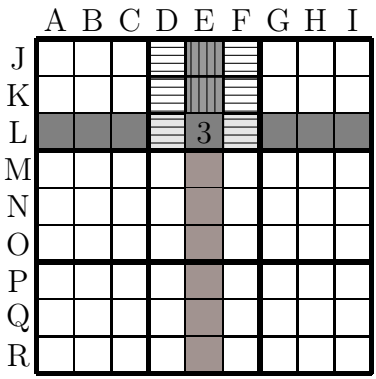
Each row, column, block, has nine cells.

A cell can contain only one number (one cell = one number). Cell FK forms at the intersection of Col F and Row K.

A particular number must be present in exactly nine cells. 5 appears exactly once in each row, column, block:
Rows: J (DJ), K (IK), L (BL), M (AM), N (HN), O (FO), P (EP), Q (GQ), R (CR).
Columns: A (AM), B (BL), C (CR), D (DJ), E (EP), F (FO), G (GQ), H (HN), I (IK).
Blocks: bl1 (BL), bl2 (DJ), bl3 (IK), bl4 (AM), bl5 (FO), bl6 (HN), bl7 (CR), bl8 (EP), bl9 (GQ).

Caveat The aim is *not* to learn to solve Sudoku (although you will inevitably do that); it is to find patterns, learn methods of analysis, goal definition, and problem solving.

If the explanation is difficult to grasp, understanding core principles and techniques involved is enough. Return to the details later, when your analytical mind has matured.



Twenty cells influence any cell,
If EL = 3, these twenty cells cannot contain 3:

- Eight cells in L (light gray).
- Eight cells in E (dark gray).
- Eight cells in bl2 (stripes).

Note:

- DL, FL belong to L, bl2.
- EJ, EK belong to E, bl2.

A Sudoku starts with some numbers already *fixed* (in place, cannot be changed), you complete the grid. A Sudoku with all 81 cells empty can be filled in 6,670,903,752,021,072,936,960 different ways!!! Try all combinations and eventually you must get a solution. But this is like monkeys randomly typing out the completed works of Shakespeare (very difficult, but possible; Book 5 explains this 'Shakespearian' problem). You can easily get one of the following correct: a row, column, or 3×3 block. Assume you start by populating a row. This is easy: populate numbers 1-9 in any of the nine cells, in any order, taking care not to repeat any number. Next try populating any column or any block, connected to the row already filled out. Sooner or later you will hit a roadblock. Putting a number in any cell will cause other dependent cells numbers to break the rules of Sudoku. A random attempt is time-consuming, and not will not guarantee a solution.
Our examples cover well-formed Sudoku's (with at least 17 numbers fixed on the grid, and a unique solution). Sudoku's with less than 17 fixed numbers may have multiple solutions.

To successfully complete a Sudoku, you need to develop strategies, know techniques, and be absolutely clear as to the rules. Use strategies and techniques outlined here (or alternates you devise or learn), but you must arrive at the same (unique) solutions illustrated in this chapter.

Legend

- Small fonts: numbers *possible* within an empty cell. One of these numbers must eventually occupy this cell.
- Large fonts: fixed numbers (that cannot be changed).
- Row: Row; Col: Column.
- Underline: Newly fixed numbers.

Examples explaining concepts may omit some fixed numbers and some possibilities, to avoid cluttering the Sudoku.

Rules/Constraints

1. Every cell can contain only one (unique) number.
2. An empty cell will eventually contain a number (1-9) not already present in the row, column, or block, to which that cell belongs (repeated numbers forbidden).
3. A number fixed in a cell can never be changed.

Strategies

1. Use fixed numbers to deduce what numbers empty cells must or must not contain.
2. More initially fixed numbers give more clues to solve the Sudoku (less possibilities). Progressing toward the solution, fewer empty cells make the goal easier.
3. Solve simplest first:
   - A smaller puzzle (nine cell blocks/rows/columns) is easier to solve than the larger (81 cells) Sudoku.
   - Tackle rows, columns, blocks, with least empty cells (almost complete) first.
4. If an empty cell can contain only one possible number, fix that number in that cell.
5. A particular number must exist in exactly nine places within a Sudoku (once in each row, column, block).
   More of a particular number present, easier to find the remaining (numbers present indicate in which cells remaining numbers can and cannot be present).

Analyzing and solving complex patterns will take time. But systematic analysis and methods of solving (techniques) will guarantee eventual success.

Basics reinforced
Break down a complex or large problem into smaller, more manageable, pieces.

Under-analysis in the real world may lead to a sub-optimal solution pathway, or failure.
With only one solution pathway available in a well-formed Sudoku, you must eventually succeed even if you initially under-analyze; provided you do not put a wrong number in any cell (*wrong analysis*).

Games with only one pathway leading to a unique outcome/ solution (for example, a well-formed Sudoku) tolerate initial under-analysis, and guarantee eventual success (provided as mentioned, you do not fix a wrong number in any cell).

Games like 'Tic, Tac, Toe' (explained on page **??**) are very unforgiving, and will not tolerate under-analysis. This is because these games have many possible pathways that can lead to dead ends (drawn games) or successful outcomes/solutions, depending on the initial moves made in the game. Forgetting to account for a possibility (under-analysis) could gift the game to the opposition.

In 'Tic, Tac, Toe', nine moves decides the game (draw, or victory for one of the players). With so few moves, initial analysis better be extremely well-thought.
Here, under-analysis (not accounting for all the possibilities) is equivalent to wrong analysis (putting a mark in the wrong cell). The outcome is usually similar, losing the game.

Do not Miss Out Any Number!

- When finding or eliminating possible numbers, try possibilities *in order* from 1-9 (or 9-1).
- Under-analysis: Even if you miss fixing a number based on obvious clues (unless you fix a number in the wrong cell), eventually you will successfully complete the Sudoku.

Understanding the techniques outlined below will take time! Understand the logic behind why each technique works. The mathematics underlining each technique is ridiculously easy, and based on logic and (mathematical) commonsense.

Techniques:

1. Possibility Matrix

	A	B	C	D	E	F	G	H	I
J						9			1
K			3					5	
L			7			2	4		6
M	8	9					1		
N	5	3		9		6		7	8
O			1		3			9	4
P	1		8	5			3		
Q		2					9		
R	7			1					

- Write all possible numbers an empty cell can contain (eliminate impossible choices).
- If a number is assigned (fixed) to a cell, remove that number as a possibility from empty cells sharing the same row, column, block.

a. Many possibilities (cannot fix)

- K contains 35; E contains 3; bl2 contains 29.
- Together they contain 2359.
- 14678 are missing (possibilities for cell EK).

b. Sole possibility (fix)

- AL belongs to A, L, bl1.
- A contains 1578; L contains 2467; bl1 contains 37.
- Together they contain 12345678 (only 9 is missing).
- AL can only contain 9 (fix AL = 9).

	A	B	C	D	E	F	G	H	I
J	12	47	149	19	1267	123	678	129	467
K			249	123	789	149			
L			159	379	179	167			
M	125	47	258	19	2367	589	368	579	129
N	249	79	389	3489	189	139			
O	126	179	38	368	68	278			
P			238	14	169	89			
Q				3567	1489	168			
R	567	134	67	39	25	169	189	47	349

c. Unique possibility (fix)

Small fonts show possible numbers that each empty cell can contain. One of these numbers must eventually be fixed in this cell.

Possible number unique to a row, or column, or block

- R: 2 is unique to ER, no other cell in the row.
  2 must be present somewhere in R.
  Only possibility is ER = 2 (fix).
  Remove 2 from bl8 (here, nothing); E (EJ, EM).
- F: 5 is unique to FM, no other cell in F (fix FM = 5).
  Remove 5 wherever present in M, bl5.
- bl8 : 7 is unique to DQ (fix DQ = 7).
  Remove 7 wherever present in Q, D.
- F has 3 in FJ, FN (cannot fix); but J has 3 only in FJ (fix FJ = 3); FJ = 3 belongs to J, F, and bl2!;
  Remove 3 from F (here, FN) and bl2 (here, DK, DL).

	A	B	C	D	E	F	G	H	I
J	12	47	149	19	1267	123	678	129	467
K			249	123	789	149			
L			159	379	179	167			
M	125	47	258	19	2367	589	368	579	129
N	249	79	389	3489	189	139			
O	126	179	38	368	68	278			
P			238	14	169	89			
Q				3567	1489	168			
R	567	134	67	39	25	169	189	47	349

d. Block-Row/Column Combination
Possible number present (twice or thrice) in cells shared by a block, *and* row/column, and nowhere else in that block.

- May be present elsewhere in the row/column.
- bl4: 3 is in CN, CO (present twice in cells shared by bl4 *and* C), no other cells of bl4.
  3 has to be present somewhere in bl4 (CN or CO).
  Wherever present (CN or CO), number 3 fulfilled for bl4 (and C).
  Remove possible 3 from other empty cells in C (here, CP).
  Cannot remove 3 from DN, FN, DO (do not know which cell, CN or CO, contains 3).
- bl4: 8 is in CM, CN, CO (on same C), and in no other cells of bl4.
  Remove possible 8 from CP.
- bl4: 5 is in AM, CM (on same M), and in no other cells of bl4.
  Remove possible 5 from FM, HM.

	A	B	C	D	E	F	G	H	I
J	1237	146	259	678	289	49	14	138	14
K		679					37	267	148
L	19	359	378	19	245	138	57	159	479
M		2458					245	26	37
N		167					16	789	489
O		69					26	137	134
P		238					123	14	357
Q		49					79	247	68
R		137					79	189	37

e. Locked Pair

Same possibility pair and nothing else, present in same...

- ...Row (but not same block), eliminate possible numbers from other cells in that row.
  AL and DL (row L) both contain same pair, 19 (AL belongs to bl1, DL belongs to bl2).
  If AL contains 1, DL must contain 9 (and vice versa). Both cells are locked.
  Remove 19 from L (9 from BL, HL, IL; 1 from FL, HL).
  BL = 359 and HL = 159 do not form a locked pair although both have 59 in common, because 5, 9 exist as possibilities in other cells of L. See *Note 4*.
- ...Column (but not same block), eliminate possible numbers from other cells in that column.
  IM and IR (column I) both contain same pair, 37 (IM belongs to bl6, IR belongs to bl9).
  Eliminate 37 from I (7 from IL; 3 from IO; 37 from IP).
- ...Block (but not same row or column), eliminate possible numbers from other cells in that block.
  HM and GO (bl6) both contain same pair, 26 (HM belongs to H and M, GO belongs to G and O).
  Eliminate 26 from bl6 (2 from GM; 6 from GN).

- . . . Row and block, eliminate possible numbers from other cells in that row *and* block.
  GJ and IJ (row J, bl3) both contain the same pair, 14.
  Eliminate 14 from J (1 from AJ, HJ; 4 from FJ; 14 from BJ) and bl3 (1 from HL; 4 from IL; 14 from IK).
- . . . Column and block, eliminate possible numbers from other cells in that column *and* block.
  GQ and GR (G, bl9) both contain the same pair, 79.
  Eliminate 79 from G (7 from GK, GL).
  Eliminate 79 from bl9 (7 from HQ, IP, IR; 9 from HR).

Note 1: In a real Sudoku, eliminating 7 from IR (because GQ and GR form a locked pair, 79) would fix IR = 3, and its corresponding locked pair cell IM = 7 (IR and IM form a locked pair, 37).

Note 2: A row and column always intersect at a single cell; locked pair for a row-column combination is impossible.

Note 3: Locked triplets (and quads) follow same principles as locked pairs. Using them though more complicated, may solve complex Sudoku's faster.

Note 4: There is a hidden locked pair in B: 28 are only present in BM, BP; nowhere else in B.
Remove possible 45 from BM, possible 3 from BP.

Note 5: After fixing a number, remember to remove possible numbers from the row, column *and* block it influences!

2. Two numbers in three blocks (2-in-3)

	A	B	C	D	E	F	G	H	I
J						9			1
K			3					5	
L	9		7			2	4		6
M	8	9					1		
N	5	3		9		6		7	8
O			1					9	4
P	1		8	5			3		
Q		2					9		
R	7			1					

Cluster groups of three adjacent horizontal or vertical blocks
Horizontal: bl1, bl2, bl3,
or bl4, bl5, bl6,
or bl7, bl8, bl9.

Vertical: bl1, bl4, bl7,
or bl2, bl5, bl8,
or bl3, bl6, bl9.

- Locate a number appearing in two blocks of a group.
- Each number will be in a separate horizontal row (*or* vertical column), and in a separate horizontal block (*or* vertical block).
- The third number must be in the horizontal row (*or* vertical column) where it is absent, and also in the horizontal (*or* vertical) block in which it is absent.

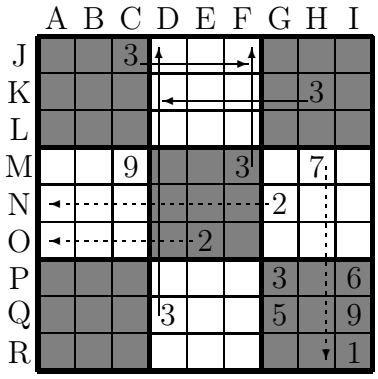
9 is present in two horizontal rows and horizontal blocks:
- L and bl1, at AL; J and bl2, at FJ.
- Must be present somewhere in K & bl3; two empty cells
  - GK: G has 9 in GQ (cannot put 9 in GK).
  - IK: I does not have 9 (fix IK = 9).

9 is present in two vertical columns and vertical blocks:
- A and bl1, at AL; B and bl4, at BM.
- Must be present somewhere in C & bl7; two empty cells
  - CQ: Q has 9 in GQ (cannot fix 9 in CQ).
  - CR: R does not have 9 (fix CR = 9).

Note1: 4 in bl3 at GL; in bl6 at IO
Somewhere in bl9 & H (HP or HQ or HR, cannot fix where)

Note 2: A missing number can only be in one of three cells. To fix which, examine the complementary blocks.
When examining horizontal bl1, bl2, bl3; to fix a number in bl3, examine complementary bl6, bl9.

3. Four block influence (4-block)
Each block has two horizontal and two vertical blocks, influencing it.



bl1: bl2, bl3, bl4, bl7.
bl2: bl1, bl3, bl5, bl8.
bl3: bl1, bl2, bl6, bl9.
bl4: bl5, bl6, bl1, bl7.
bl5: bl4, bl6, bl2, bl8.
bl6: bl4, bl5, bl3, bl9.
bl7: bl8, bl9, bl1, bl4.
bl8: bl7, bl9, bl2, bl5.
bl9: bl7, bl8, bl3, bl6.

Note the missing numbers for a given block:

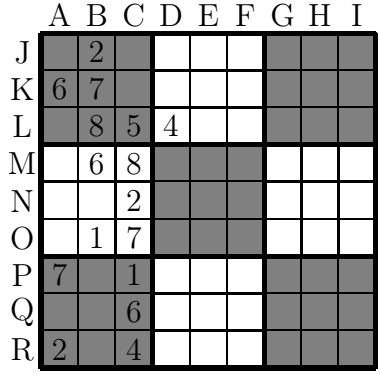
- Draw imaginary lines from each number present in the influencing blocks (but missing in the given block), up to the given block.
- Eliminate empty cells the imaginary lines pass through (the missing number cannot occupy these cells).
- If only one empty cell remains in a given block, fix that missing number in that cell.
- Between one and four blocks influence the given block.
- Similar to Technique 1 (Possibility Matrix), but the Sudoku is not cluttered with possible numbers.
- This technique works for simple Sudoku's; for complex ones, use the Possibility Matrix (Technique 1).

bl2: All except one cell eliminated by 3 in bl1, bl3, bl5, bl8 (fix EL = 3).

bl4: 2 can occupy AM or BM (cannot fix). Only two blocks (bl5, bl6) contain a '2' to influence this (bl4) block.

bl9: One 7 (in bl6) is enough to fix GR = 7 in bl9. Here, a single influencing block (bl6) contains a '7'.

4. Swipe
Three cells at the intersection of a row (or column) and block are influenced by the other six cells of that block.



A and intersection of. . .
. . . bl1: BL = 8; swipes 8 from AJ, AL.
. . . bl4: CM = 8; swipes 8 from AM, AN, AO.
. . . bl7: only AQ free; AQ=8.

C has 39 missing: If CJ = 3, CK = 9, and vice versa.

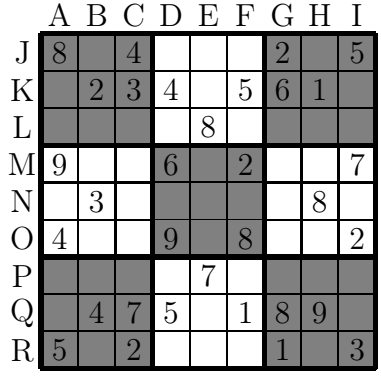
bl1 has 1349 missing.
39 accounted for (in CJ, CK), 14 must be in AJ or AL.
DL = 4; so AL = 1, AJ = 4.

Technique 1 (and sub-techniques b, c, d, e) is intuitive to understand but tedious to use. It involves carefully writing all possible numbers a cell can contain, making it effort- and error-prone! Use it last, if simpler techniques fail.
Technique 2 is the easiest and most efficient to start solving simple Sudoku's (followed by Technique 3). Techniques 2, 3 are similar to Technique 1 (but infinitely faster and easier!).
Technique 4 is actually a clever trick. With experience, you will discover more of these shortcuts yourself.
One size does not fit all To solve a given Sudoku, you will have to use multiple techniques, or invent your own ones.

Mathematicians and experts use efficient (though difficult and complicated to understand) techniques. We use simple to understand techniques that solve any Sudoku, and fulfill our primary aim (teach problem solving, not puzzle solving).

Just do it! Do not mentally visualize steps outlined here. The Sudoku cannot be redrawn after each clue is explained. Create the initial (problem) Sudoku, on paper. As each clue is explained, fill the appropriate empty cells. When the Sudoku is redrawn, newly fixed numbers will be underlined.

A systematic approach starts our simple Sudoku quest.



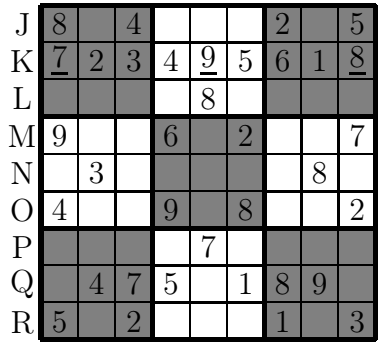
	A	B	C	D	E	F	G	H	I
J	8		4				2		5
K		2	3	4		5	6	1	
L					8				
M	9			6		2			7
N		3						8	
O	4			9		8			2
P					7				
Q		4	7	5		1	8	9	
R	5		2				1		3

K has only three empty cells 789 missing in AK, EK, IK.

AK ≠ 8: AJ = 8 and AK belong to same A, same bl1.
AK ≠ 9: AM = 9 and AK belong to same A.
AK = 7: Last alternative.

Only 89 remain for K.
EK ≠ 8: EL = 8 and EK belong to same E, same bl2.

EK = 9: Last choice; IK = 8: Last number remaining in K.



	A	B	C	D	E	F	G	H	I
J	8		4				2		5
K	7	2	3	4	9	5	6	1	8
L					8				
M	9			6		2			7
N		3						8	
O	4			9		8			2
P					7				
Q		4	7	5		1	8	9	
R	5		2				1		3

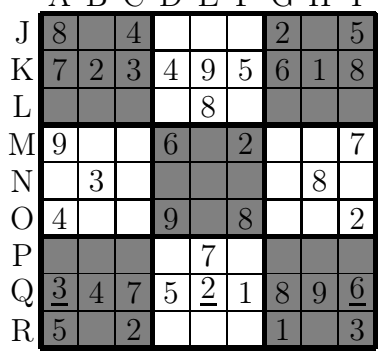
Put three new numbers in K.

Q: 236 absent in AQ, EQ, IQ.
AQ ≠ 2: CR in same bl7.
AQ has 3 or 6; AQ = (36).

EQ = (236): Nothing to eliminate.

IQ ≠ 2: IO in same I.
IQ ≠ 3: IR in same I & bl9.
IQ = 6: Last alternative.

Now, AQ = 3; EQ = 2 (only number remaining in Q).



	A	B	C	D	E	F	G	H	I
J	8		4				2		5
K	7	2	3	4	9	5	6	1	8
L					8				
M	9			6		2			7
N		3						8	
O	4			9		8			2
P					7				
Q	3	4	7	5	2	1	8	9	6
R	5		2				1		3

Tackle empty cells in A & I.
126 absent in AL, AN, AP.
AL = (16); AN = (126);
AP = (16).
Cannot fix A's cells.

149 missing from IL, IN, IP.
IL = (49); IN = (149); IP = 4.
IL = 9; IN = 1 (only number remaining).

Recap: We tackled rows and columns with least empty cells, then looked at possible numbers a cell could contain.

	A	B	C	D	E	F	G	H	I
J	8		4				2		5
K	7	2	3	4	9	5	6	1	8
L					8				9
M	9			6		2			7
N		3						8	1
O	4			9		8			2
P					7				4
Q	3	4	7	5	2	1	8	9	6
R	5		2				1		3

bl3, bl9, have three empty cells each.
bl3: 347 missing from HJ, GL, HL.
HJ = (37); GL = (347); HL = (347).
bl9: 257 missing from GP, HP, HR.
GP = 5 (fix).

HP = 2 (since GP = 5 now), fix.
HR = 7 (only number remaining), fix.
Now, bl3: HJ = 3; HL = 4; GL = 7 (last number in bl3).

As an exercise, continue using this technique on other rows, columns, blocks. We move on, to learn a new technique.

	A	B	C	D	E	F	G	H	I
J	8		4				2	3	5
K	7	2	3	4	9	5	6	1	8
L					8		7	4	9
M	9			6		2			7
N		3						8	1
O	4			9		8			2
P					7		5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5		2				1	7	3

2-in-3 technique for bl1, bl2, bl3 (in rows).
16 present once: insufficient information.
48 present thrice: nothing to find.
23579 present twice: find the third.

2 in BK (so absent elsewhere in K & bl1), GJ (so absent elsewhere in J & bl3):
Search cells at intersection of L & bl2 (DL & FL are empty).
FL ≠ 2 as FM = 2, DL = 2 (fix).
3 in CK (K, bl1), HJ (J, bl3): FL = 3 (L, bl2) as DL = 2.
5 in FK (K, bl2), IJ (J, bl3); L, bl1: not in AL (AR = 5); in BL or CL (two choices, cannot fix).
7 in AK (K, bl1), HL (L, bl3): J, bl2, not in EJ (EP = 7); in DJ or FJ (two choices, cannot fix).
9 in EK (K, bl2), IL (L, bl3): BJ = 9 (J, bl1), last empty cell.

	A	B	C	D	E	F	G	H	I
J	8	9	4				2	3	5
K	7	2	3	4	9	5	6	1	8
L				2	8	3	7	4	9
M	9			6		2			7
N		3						8	1
O	4			9		8			2
P					7		5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5		2				1	7	3

Two located, find third for bl4, bl5, bl6 (in rows):

2 in M (FM) & bl5, O (IO) & bl6: Find in N & bl4.
CN ≠ 2 (CR = 2); AN = 2.

8 in O (FO) & bl5, N (HN) & bl6: Find in M & bl4.
Can be in BM or CM, (two choices, cannot fix).

2-in-3 for bl7, bl8, bl9 (in rows):

- 1 in Q & bl8, R & bl9: In P & bl7, can be in AP, BP, CP (cannot fix).
- 3 in Q & bl7, R & bl9: In P & bl8, DP = 3 (FL = 3).
- 4 in Q & bl7, P & bl9: In R & bl8, can be in ER or FR (two choices, cannot fix).

	A	B	C	D	E	F	G	H	I
J	8	9	4				2	3	5
K	7	2	3	4	9	5	6	1	8
L				2	8	3	7	4	9
M	9			6		2			7
N	2	3						8	1
O	4			9		8			2
P				3	7		5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5		2				1	7	3

2-in-3, columns: bl1, bl4, bl7

- 7 in A & bl1, C & bl7: In B & bl4, BO = 7.
- 9 in B & bl1, A & bl4: In C & bl7, CP = 9.

2-in-3, columns: bl2, bl5, bl8

- 3 in EM or EO.
- 5 in EM or EN or EO.
- DR = 8.
- CP = 9 from 'bl1, bl4, bl7' above, so FR = 9.

From bl7, bl8, bl9, 4 was present in ER or FR.
Since FR = 9 (from above), ER = 4.

2-in-3 for bl3, bl6, bl9 (columns)

- 3 in GM or GO; 4 in GM or GN.
- 5 in HM or HO; HO = 6 (so, HM = 5).
- GN = 9; so, GM = 4; so, GO = 3.

Note: We failed to account for GN = 9 way back in 2-in-3 for bl4, bl5, bl6 (in rows)! No harm done, fixed now. This is what we implied on page 5, when we said under-analysis would not be penalized. You will eventually get the correct solution. If however you err in the analysis and fix the wrong number in a cell, you are guaranteed to fail.

	A	B	C	D	E	F	G	H	I
J	8	9	4				2	3	5
K	7	2	3	4	9	5	6	1	8
L				2	8	3	7	4	9
M	9			6		2	4	5	7
N	2	3					9	8	1
O	4	7		9		8	3	6	2
P			9	3	7		5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5		2	8	4	9	1	7	3

Put identified numbers to finish the 2-in-3 technique.
One number missing in R and bl8: BR = 6; FP = 6.

Two numbers missing in bl7: AJ = 8, so AP = 1; now BP = 8.
Missing in A: AL = 6.

2-in-3 is finished for rows and columns, but a second round will help to fix more values.

	A	B	C	D	E	F	G	H	I
J	8	9	4				2	3	5
K	7	2	3	4	9	5	6	1	8
L	6			2	8	3	7	4	9
M	9			6		2	4	5	7
N	2	3					9	8	1
O	4	7		9		8	3	6	2
P	1	8	9	3	7	6	5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5	6	2	8	4	9	1	7	3

bl2 has 167 missing: 1 in DJ or EJ; EJ = 6, so DJ = 1; now FJ = 7.

D & F miss one number each: DN = 7; FN = 4.

B is missing two numbers: BM = 1 (HM = 5); now BL = 5.

2-in-3 for bl4, bl5, bl6: BM = 1, so CM = 8.
L, M are missing one number, CL = 1, EM = 3.

	A	B	C	D	E	F	G	H	I
J	8	9	4	1	6	7	2	3	5
K	7	2	3	4	9	5	6	1	8
L	6	5	1	2	8	3	7	4	9
M	9	1	8	6	3	2	4	5	7
N	2	3		7		4	9	8	1
O	4	7		9		8	3	6	2
P	1	8	9	3	7	6	5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5	6	2	8	4	9	1	7	3

C is missing 5, 6:

- CO ≠ 6 (HO = 6).
- CO = 5; CN = 6.

E is missing 1, 5:

- EN ≠ 1 (IN = 1).
- EN = 5; EO = 1.

Or check N, O for missing numbers.

	A	B	C	D	E	F	G	H	I
J	8	9	4	1	6	7	2	3	5
K	7	2	3	4	9	5	6	1	8
L	6	5	1	2	8	3	7	4	9
M	9	1	8	6	3	2	4	5	7
N	2	3	6	7	5	4	9	8	1
O	4	7	5	9	1	8	3	6	2
P	1	8	9	3	7	6	5	2	4
Q	3	4	7	5	2	1	8	9	6
R	5	6	2	8	4	9	1	7	3

The Solution!
Perhaps you arrived at this unique solution via a different solution pathway. No problem! The more pathways (exciting and clever tricks to get to the goal) you discover, the more patterns and connections you will pick up.

General Observations

- A missing number can be located using multiple techniques; any technique that works for the Sudoku, *must* give the same number in the same cell.
- Whichever technique used to find (and fix) numbers, the final solution is unique.
- More the amount of a given number present initially, easier it becomes to find the remaining numbers (since each number only appears nine times in a Sudoku).
- Under-analysis is tolerated (here); wrong analysis results in failure.

Now, a *Super-tough* Sudoku!

	A	B	C	D	E	F	G	H	I
J		6					1	4	
K		4			7				
L	1			4	2	6			3
M		1	2			7	5		
N					4		3		
O			6	2			8	9	
P	7		1	3	8		6	5	9
Q					6			1	8
R	6	8	5					3	

P has 24 missing: BP = 2 (as BK = 4); FP = 4.

Fill P.

Easy techniques (2-in-3, 4-block) do not work.

Put *all* possible values in vacant cells.

Even with a super-tough Sudoku, you may be able to (partially) use super-easy techniques.

	A	B	C	D	E	F	G	H	I
J	23589	6	3789	589	359	3589	1	4	257
K	23589	4	389	1589	7	13589	29	268	256
L	1	579	789	4	2	6	79	78	3
M	3489	1	2	689	39	7	5		
N					4		3		
O			6	2			8	9	
P	7	2	1	3	8	4	6	5	9
Q					6			1	8
R	6	8	5					3	

HM can only contain 6. Remove *possible* 6 from M, H, bl6. Because HM = 6, IM = 4. Remove *possible* 4 from M, I, bl6. Continue putting all possible values in empty cells.

	A	B	C	D	E	F	G	H	I
J	23589	6	3789	589	359	3589	1	4	257
K	23589	4	389	1589	7	13589	29	28	256
L	1	579	789	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	589	579	789	15689	4	1589	3	27	127
O	345	357	6	2	135	135	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	349	39	349	579	6	259	247	1	8
R	6	8	5	179	19	129	247	3	27

Fix numbers unique in row, column, block. Remove that *possible* number from the associated row, column, block.

Numbers unique in rows:
6 must be somewhere in K, only IK is possible.
Fix IK = 6. Remove *possible* 6 from I, bl3 (here, nothing).

Fix BL = 5 (5 is unique in L). Remove *possible* 5 from AJ, AK (bl1); BN, BO (L).

Fix DN = 6 (6 is unique in N *and* D *and* bl5).

Fix AO = 4 (4 is unique in O *and* bl4). Remove *possible* 4 from AQ (A).

Fix GR = 4 (4 is unique in R). Remove *possible* 4 from GQ (G *and* bl9).

	A	B	C	D	E	F	G	H	I
J	2389	6	3789	589	359	3589	1	4	257
K	2389	4	389	1589	7	13589	29	28	6
L	1	5	789	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	589	79	789	6	4	1589	3	27	127
O	4	37	6	2	135	135	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	349	579	6	259	27	1	8
R	6	8	5	179	19	129	4	3	27

Numbers unique in columns:
Fix AN = 5 (5 is unique in A *and* bl4). Remove *possible* 5 from FN (N).

Fix CQ = 4 (4 is unique in C *and* Q *and* bl7).

Fix IJ = 5 (5 is unique in I *and* bl3). Remove *possible* 5 from DJ, EJ, FJ (J).

EO = (135) and FO = (135) do not form a locked pair! They may have formed a locked triplet if another (135) existed in an empty cell of either O or bl5.

AQ = (39) and BQ = (39) form a locked pair in Q and bl7. This removes (39) from all empty cells of Q. Effectively, CQ = 4 (also found from above).

GQ = (27) and IR = (27) form a locked pair in bl9.

	A	B	C	D	E	F	G	H	I
J	2389	6	3789	89	39	389	1	4	5
K	2389	4	389	1589	7	13589	29	28	6
L	1	5	789	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	5	79	789	6	4	189	3	27	127
O	4	37	6	2	135	135	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	4	579	6	259	27	1	8
R	6	8	5	179	19	129	4	3	27

AQ = BQ = (39) form a locked pair in Q, bl7 (both belong to the same row, same block).
Eliminate (39) from other empty cells of Q (DQ, FQ), bl7 (here, nothing).

GQ = IR = (27) form a locked pair in bl9: same block but not same row or column.
Eliminate 27 from other cells of bl9 (here, nothing).

Fix AJ = 2 (2 is unique in J). Remove *possible* 2 from AK (A, bl1).

	A	B	C	D	E	F	G	H	I
J	2	6	3789	89	39	389	1	4	5
K	389	4	389	1589	7	13589	29	28	6
L	1	5	789	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	5	79	789	6	4	189	3	27	127
O	4	37	6	2	135	135	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	4	57	6	25	27	1	8
R	6	8	5	179	19	129	4	3	27

C: 7 is present in CJ, CL, and nowhere else in bl1; has to be present in one of these two cells (to fulfill bl1 quota). Eliminate *possible* 7 from CN.

L: 7 is present in GL, HL, and nowhere else in bl3; has to be present in one of these two cells (to fulfill bl3 quota). Eliminate *possible* 7 from CL.

Now C has 7 only in CJ, fix CJ = 7.

Now C has 3 only in CK, fix CK = 3. Remove *possible* 3 from AK (K, bl1) and FK (K).

	A	B	C	D	E	F	G	H	I
J	2	6	7	89	39	389	1	4	5
K	89	4	3	1589	7	1589	29	28	6
L	1	5	89	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	5	79	89	6	4	189	3	27	127
O	4	37	6	2	135	135	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	4	57	6	25	27	1	8
R	6	8	5	179	19	129	4	3	27

Fix EO = 5 (5 is unique in E). Remove *possible* 5 from FO (belongs to O and bl5).

Now, 1 must be present in FN or FO (quota of bl5). Remove *possible* 1 from FK, FR (all four cells belong to same F).
This forces DK = 1 (unique in bl2), which in turn forces ER = 1 (unique in bl8).

Remove all *possible* numbers that are now impossible.

	A	B	C	D	E	F	G	H	I
J	2	6	7	89	39	389	1	4	5
K	89	4	3	1	7	589	29	28	6
L	1	5	89	4	2	6	79	78	3
M	389	1	2	89	39	7	5	6	4
N	5	79	89	6	4	189	3	27	127
O	4	37	6	2	5	13	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	4	57	6	25	27	1	8
R	6	8	5	79	1	29	4	3	27

DQ = 5 (unique in D). FQ = 2 (now unique in FQ).
FR = 9 (now unique in FR). DR = 7 (now unique in DR).
GQ = 7 (now unique in Q). IR = 2 (now unique in IR).

	A	B	C	D	E	F	G	H	I
J	2	6	7	89	39	38	1	4	5
K	89	4	3	1	7	58	29	28	6
L	1	5	89	4	2	6	9	78	3
M	389	1	2	89	39	7	5	6	4
N	5	79	89	6	4	18	3	27	127
O	4	37	6	2	5	13	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	39	39	4	5	6	2	7	1	8
R	6	8	5	7	1	9	4	3	2

FK = 5 (5 is unique in F, K, bl2). GL = 9 (unique in GL).
Now GK = 2, then HK = 8, then AK = 9, then CL = 8, then HL = 7 (each *possible* number is now unique in its cell).

Possible numbers unique in cells:

CN = 9. Now BN = 7.

HN = 2. Now IN = 1. Now FN = 8.

AQ = 3. Now AM = 8 and BQ = 9.

	A	B	C	D	E	F	G	H	I
J	2	6	7	89	39	38	1	4	5
K	9	4	3	1	7	5	2	8	6
L	1	5	8	4	2	6	9	7	3
M	38	1	2	89	39	7	5	6	4
N	5	79	9	6	4	18	3	2	127
O	4	37	6	2	5	13	8	9	17
P	7	2	1	3	8	4	6	5	9
Q	3	39	4	5	6	2	7	1	8
R	6	8	5	7	1	9	4	3	2

Follow any pathway to a unique solution. One pathway is:

FJ = 3, EJ = 9, DJ = 8.

DM = 9, EM = 3, FO = 1.

BO = 3, IO = 7.

	A	B	C	D	E	F	G	H	I
J	2	6	7	89	39	3	1	4	5
K	9	4	3	1	7	5	2	8	6
L	1	5	8	4	2	6	9	7	3
M	8	1	2	9	39	7	5	6	4
N	5	7	9	6	4	8	3	2	1
O	4	3	6	2	5	13	8	9	7
P	7	2	1	3	8	4	6	5	9
Q	3	9	4	5	6	2	7	1	8
R	6	8	5	7	1	9	4	3	2

Unique Solution!
No matter which technique you used and what number you missed putting in a cell (provided you did not put the wrong number in a given cell), you *must* get this solution.

	A	B	C	D	E	F	G	H	I
J	2	6	7	8	9	3	1	4	5
K	9	4	3	1	7	5	2	8	6
L	1	5	8	4	2	6	9	7	3
M	8	1	2	9	3	7	5	6	4
N	5	7	9	6	4	8	3	2	1
O	4	3	6	2	5	1	8	9	7
P	7	2	1	3	8	4	6	5	9
Q	3	9	4	5	6	2	7	1	8
R	6	8	5	7	1	9	4	3	2

Techniques to solve Sudoku efficiently is a topic worthy of research. We stop here, with our goal of learning problem solving accomplished. Solve more Sudoku's, discover your own hidden patterns and clever solving techniques.

The more Sudoku's you solve, the more experience you gain. This will develop your logic, your ability to dig out patterns and make connections with the given information. Patterns you find and connections you make in Sudoku, relate to numbers. But the underlying logic and understanding of patterns and connections is universal for numbers, letters, or indeed any collection of objects, elements, information.

Modified Sudoku puzzles can change the pattern or the grid size; or replace the numbers with letters or exotic characters or patterns or parts of an image. Once you understand the logic of what is required and what is not allowed (we call this analysis), you will find guaranteed techniques for success.

Many games in this chapter (starting with the next one, Solitaire) can only be explained from a problem solving standpoint by explaining scenarios. There is nothing for you, the reader, to solve. Understand the explanations and use the logic developed in the book, to conjure up (and solve) fresh gaming situations on your own.

Please send feedback! `info@problemsolvingpathway.com`

'Sudoku Strategy' is part of a chapter on learning problem solving using games.

Download the free book of 'selected problems', available on the website.

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To Get Lost Is To Find The Pathway

This book series teaches core problem solving concepts: Definition, exploration, sensing and verifying detail (via observation and analysis), separating fact from fiction, retaining relevant facts, thinking (logically, analytically, critically), stated versus implied information, investigating all possible solution paths while abandoning impossible ones, educated guesswork, understanding what is an acceptable solution, judging and deciding, implementing the optimal solution, learning from failure or inefficient solutions, seeking feedback, re-implementing a better solution.

Each self-contained book teaches a separate set of concepts. The problem solving apprentice is advised to sequentially follow the learning pathway, the expert can enhance specific problem solving concepts.

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V: Real World Tools Thinking, questioning, attention to relevant detail, intuition, judging & deciding, solving real world problems.

VI: Along The Pathway Defining the problem solving process, awareness of obstacles to problem solving, tips for the ace problem solver.

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